Spacetime from Self-Modeling: Einstein's Equations from a Self-Modeling Lattice via Jacobson's Thermodynamic Argument

Bryan Ehrlich[1]

[1] *Independent researcher*

(Dated: March 23, 2026)

We show that the locality of self-modeling—the principle that a physical system contains a faithful model of itself, updated through its boundary—yields Einstein's field equations as the leading-order effective description at long wavelengths, given lattice topology and antiferromagnetic coupling as inputs. Self-modeling forces $M_n(\mathbb{C})^{sa}$ quantum mechanics [1]. The self-modeling interaction Hamiltonian $H = \sum_{\langle x,y\rangle} J\, F_{xy}$, forced by diagonal $U(n)$ covariance, produces area-law entanglement entropy $S = \eta\,\mathcal{A}$. Applying Jacobson's thermodynamic argument [2]—the Clausius relation $\delta Q = T\,\delta S$ on local Rindler horizons, with the Unruh temperature and the Raychaudhuri equation—yields

$$G_{ab} + \Lambda\, g_{ab} = 8\pi G\, T_{ab}$$

with $G = 1/(4\eta)$, in all $d+1 \geq 3$ spacetime dimensions. We present numerical verification on small lattices ($N = 8$–20) and precisely identify remaining gaps: emergent Lorentz invariance, the lattice Bisognano–Wichmann property, and the local equilibrium condition.

I. INTRODUCTION

Self-modeling—the condition that a physical system contains a faithful model of itself, updated through its boundary—forces the algebraic structure of quantum mechanics [1]. We show that it also forces general relativity. Specifically, Einstein's field equations emerge as the leading-order infrared effective description of a lattice of self-modeling systems in the Wilsonian continuum limit.

The idea that spacetime geometry is encoded in quantum entanglement has a rich history. Van Raamsdonk argued that entanglement is the glue that holds spacetime together [3]. Jacobson showed that the Einstein equation can be derived as an equation of state from the Clausius relation $\delta Q = T\,\delta S$ applied to local Rindler horizons [2]. Lashkari, McDermott, and Van Raamsdonk demonstrated that the entanglement first law in holographic conformal field theories implies the linearized Einstein equation [4], and Faulkner *et al.* extended this to the full nonlinear equations in the holographic setting [5]. Cao, Carroll, and Michalakis showed that spatial geometry can be recovered from the entanglement structure of a finite-dimensional Hilbert space [6].

These results leave open a central question: *what ultraviolet completion produces the entanglement structure that Jacobson's argument requires?* The holographic results of [4, 5] assume AdS/CFT; the Jacobson argument [2] is UV-agnostic. We provide a specific, non-holographic UV completion: a lattice of self-modeling quantum systems whose interaction Hamiltonian is forced by the algebraic structure of quantum theory itself.

The derivation proceeds in a single chain. Self-modeling is the sole physical premise. The lattice topology $G = (V, E)$ and the sign of the coupling ($J > 0$, antiferromagnetic) are inputs. The forced Hamiltonian produces area-law entanglement entropy $S = \eta\,\mathcal{A}$. The Wilsonian continuum limit yields an emergent smooth manifold with emergent Lorentz invariance. The Bisognano–Wichmann theorem identifies the modular flow with local Lorentz boosts, giving the Unruh temperature. The Clausius relation $\delta Q = T\,\delta S$ on local Rindler horizons, combined with the Raychaudhuri equation, then yields Einstein's equations [2].

This paper does not derive Newton's constant G (which depends on the lattice entropy density η via $G = 1/(4\eta)$), the spacetime dimension (which is set by the lattice topology), or the cosmological constant Λ (which appears as an undetermined integration constant). The sign chain producing attractive gravity assumes the null energy condition (NEC).

The complete derivation chain, from self-modeling to Einstein's equations, is summarized in Table I.

The novel contribution of this work is the assembly of Links L1–L5 as a UV completion, with L2–L4 being new results: the forced Hamiltonian (L2), emergent causal structure (L3), and area-law bounds (L4). L1 is established in Paper 5 [1] and L5 is a standard identity of quantum information theory. Links L6–L8 are standard bridge assumptions (continuum limit, Lorentz invariance, Bisognano–Wichmann). Link L9 applies Jacobson's 1995 thermodynamic argument [2] to convert the UV inputs into Einstein's equations.

The paper is organized as follows. Section II defines the self-modeling lattice: the local algebra $M_n(\mathbb{C})^{sa}$ at each site, the SWAP interaction Hamiltonian forced by diagonal $U(n)$ covariance, and the emergent causal structure from the Lieb-Robinson velocity [8, 9]. Section III establishes sub-volume entanglement scaling: the WVCH thermal mutual information bound [10], known ground-state entanglement properties of the Heisenberg model [11, 12], and the entanglement first law combined with modular Hamiltonian locality. Section IV presents the Wilsonian continuum limit, emergent Lorentz invariance, the Bisognano–Wichmann identification of modu-

TABLE I. Derivation chain from self-modeling to Einstein's equations. "Status" gives the logical standing of each link. "Novel?" indicates what is new in this work versus prior results.

Link	Statement	Status	Section	Novel?	Key input
L1	Self-modeling forces $M_n(\mathbb{C})^{sa}$ with Lüders product	Proven	—	Ref. [1]	None
L2	Lattice Hamiltonian $H = \sum J\, F_{xy}$ forced by $U(n)$ covariance	Derived	II	This work	Lattice topology,
L3	Lieb-Robinson velocity v_{LR} gives emergent causal structure	Derived	II	This work	L2
L4	Area-law entanglement entropy $S = \eta\,\mathcal{A}$	Supported	III	This work	L2, L3
L5	Entanglement first law: $\delta S = \delta\langle K_A\rangle$	Exact identity	III	Standard QI	None
L6	Wilsonian continuum limit	Standard methodology	IV A	Standard	L2
L7	Emergent Lorentz invariance	Expected (Gap 1)	IV A	Standard	L6
L8	Unruh temperature from Bisognano–Wichmann	Standard + Gap 2	IV B	Standard [7]	L7
L9	$G_{ab} + \Lambda\, g_{ab} = 8\pi G\, T_{ab}$ via Jacobson (1995)	Derived	IV D	Jacobson [2] applied	L4, L7, L8

lar flow with local boosts, and the Jacobson (1995) thermodynamic derivation of Einstein's equations. Section V presents numerical verification on small lattices ($N = 8$–20): exact diagonalization benchmarks, area-law scaling, and modular Hamiltonian locality as evidence for the lattice Bisognano–Wichmann property. Section VI identifies remaining gaps, compares with related work, and outlines future directions.

II. THE SELF-MODELING LATTICE

A. Self-modeling forces quantum mechanics

Paper 5 [1] established that faithful self-modeling—the condition that a physical system B contains a subsystem M that tracks the state of B through their shared boundary—forces the local state space to be $M_n(\mathbb{C})^{sa}$, the self-adjoint part of the $n \times n$ complex matrix algebra, equipped with the Lüders sequential product

$$a \,\&\, b = a^{1/2}\, b\, a^{1/2}. \tag{1}$$

The real and quaternionic alternatives are excluded by local tomography: only complex quantum mechanics has $\dim(V_{BM}) = \dim(V_B)\cdot\dim(V_M)$. The sequential product is the unique product compatible with axioms S1–S7 of Ref. [13], and it inherits $U(n)$ covariance from the self-modeling structure. We do not re-derive these results here; they serve as the starting point for the present paper.

B. Lattice definition

We place self-modeling systems on a lattice described by an undirected graph $G = (V, E)$, where V is a countable set of sites and $E \subseteq \binom{V}{2}$ is the set of edges encoding nearest-neighbor adjacency. At each site $x \in V$ the local algebra is

$$A_x = M_n(\mathbb{C}), \tag{2}$$

and for a finite region $\Lambda \subset V$ the regional algebra is

$$A_\Lambda = \bigotimes_{x\in\Lambda} A_x \cong M_{n^{|\Lambda|}}(\mathbb{C}). \tag{3}$$

The quasi-local algebra $\mathfrak{A} = \overline{\bigcup_{|\Lambda|<\infty} A_\Lambda}^{\|\cdot\|}$ is a uniformly hyperfinite (UHF) C*-algebra of type n^∞.

Background dependence.—The graph topology $G = (V, E)$ is an irreducible *input* to this construction. Self-modeling determines the metric (encoded in the coupling strengths and the resulting entanglement structure) on a given topology, but not the topology itself. This is analogous to general relativity, where Einstein's equations determine the metric on a given differentiable manifold without determining its topology. The spatial dimension d is set by the graph structure: a d-dimensional regular lattice yields d emergent spatial dimensions.

C. Hamiltonian construction

The interaction between adjacent sites is encoded in a two-site Hamiltonian $h_{xy} \in A^{sa}_{\{x,y\}}$ for each edge $\{x, y\} \in E$. Self-modeling locality—the constraint that model and body interact only through their shared boundary—implies that $h_{xy} = 0$ for non-adjacent pairs ($\{x, y\} \notin E$), mapping self-modeling locality directly to nearest-neighbor Hamiltonian locality [1].

The Lüders sequential product (1) is covariant under the adjoint action of $U(n)$:

$$U(a \,\&\, c)U^\dagger = (UaU^\dagger) \,\&\, (UcU^\dagger) \qquad \forall\, U \in U(n). \tag{4}$$

The composite sequential product $(a\otimes b)\,\&\,(c\otimes d) = (a\,\&\, c)\otimes(b\,\&\, d)$ is covariant under $U(n)\times U(n)$ (independent rotations at each site). The coupling, however, physically connects the two sites through a shared boundary; basis independence requires that the same rotation be applied to both sites. This constrains h_{xy} to be invariant under the *diagonal* $U(n)$:

$$(U \otimes U)\, h_{xy}\, (U \otimes U)^\dagger = h_{xy} \qquad \forall\, U \in U(n). \tag{5}$$

Full $U(n) \times U(n)$ invariance would kill all coupling, leaving $h_{xy} \propto \mathbb{1} \otimes \mathbb{1}$.

By **Schur–Weyl duality** for the symmetric group S_2 acting on $(\mathbb{C}^n)^{\otimes 2}$, this space decomposes under the diagonal $U(n)$ action into two irreducible representations: $\mathrm{Sym}^2(\mathbb{C}^n) \oplus \wedge^2(\mathbb{C}^n)$. By Schur's lemma the commutant of $\{U \otimes U\}$ is $\mathrm{span}\{\mathbb{1}, F\}$, where F is the SWAP operator defined by

$$F_{xy}|i\rangle|j\rangle = |j\rangle|i\rangle. \tag{6}$$

Therefore the most general diagonal-$U(n)$-invariant self-adjoint interaction is

$$h_{xy} = \alpha\, \mathbb{1} \otimes \mathbb{1} + J\, F_{xy}, \tag{7}$$

with $\alpha, J \in \mathbb{R}$. The identity term shifts the energy zero; setting $\alpha = 0$, the total lattice Hamiltonian is

$$\boxed{H = \sum_{\langle x,y\rangle \in E} J\, F_{xy}.} \tag{8}$$

The interaction is *forced*: the form $h_{xy} = J\, F_{xy}$ is the unique diagonal-$U(n)$-invariant coupling (up to the overall energy scale J). Any anisotropy would break the $U(n)$ covariance inherited from the Lüders product. For $n = 2$, the SWAP operator takes the familiar form $F_{xy} = \frac{1}{2}(\mathbb{1} \otimes \mathbb{1} + \vec{\sigma} \cdot \vec{\sigma})$, and the Hamiltonian (8) reduces to the isotropic Heisenberg model.

The sign of J is *not* determined by the self-modeling constraints: $J > 0$ gives the antiferromagnetic (AFM) Heisenberg model, $J < 0$ gives the ferromagnetic (FM) model. Both signs are compatible with the algebraic structure. However, the FM ground state ($J < 0$) is a product state with $S(A) = 0$ for all subsystem sizes, producing no entanglement structure and hence no emergent geometry. Only the AFM case ($J > 0$) supports the nontrivial entanglement that drives the Jacobson argument. We therefore work with $J > 0$ throughout; this is an input to the derivation, not forced by self-modeling.

D. Lieb–Robinson bounds and emergent causal structure

The nearest-neighbor Hamiltonian (8) with finite coupling $\|h_{xy}\| = |J|$ satisfies the hypotheses of the Lieb–Robinson theorem [8, 9]. For any local observables A_x and B_y supported at sites x and y separated by graph distance $d(x, y)$, the Heisenberg-picture commutator is bounded by

$$\|[\tau_t(A_x),\, B_y]\| \le C_{\mathrm{LR}}\, \|A\|\, \|B\|\, e^{-\mu(d(x,y) - v_{\mathrm{LR}}|t|)}, \tag{9}$$

where $\tau_t(A) = e^{iHt} A\, e^{-iHt}$ is the Heisenberg evolution, $\mu > 0$ is a spatial decay constant, C_{LR} is a dimension-dependent prefactor, and v_{LR} is the **Lieb–Robinson velocity**—the maximum speed of information propagation on the lattice.

For the self-modeling Hamiltonian on a lattice with coordination number z, the Nachtergaele–Sims framework [9] with exponential decay function $F_a(r) = e^{-ar}$ at $a = 1$ gives

$$v_{\mathrm{LR}} = \frac{4ze|J|}{e - 1}. \tag{10}$$

For the one-dimensional chain ($z = 2$), this is $v_{\mathrm{LR}} = 8e|J|/(e-1) \approx 12.66\,|J|$. Crucially, v_{LR} is *independent of the local dimension* n: the SWAP operator F satisfies $\|F\| = 1$ for all $n \ge 2$, so the Lieb–Robinson velocity depends only on the coupling strength $|J|$ and the lattice coordination number z.

The bound (9) defines an effective *light cone* on the lattice: correlations outside the cone ($d(x, y) > v_{\mathrm{LR}}|t|$) are exponentially suppressed. In the continuum limit (Sec. IV), v_{LR} is identified with the emergent speed of light c.

Causal structure, not Lorentz invariance.—The Lieb–Robinson bound provides an emergent *causal structure*—a finite maximum signaling speed—but not Lorentz invariance. The lattice has a preferred frame (the rest frame of the lattice). Lorentz invariance of the low-energy effective theory is an output of the Wilsonian continuum limit (Sec. IV A), not a property of the lattice itself. The Lieb–Robinson velocity v_{LR} sets the *scale* of the emergent speed of light; the *symmetry* (Lorentz covariance) is a property of the long-wavelength effective theory, realized when lattice artifacts are suppressed at $L \gg a$.

III. AREA-LAW ENTANGLEMENT

The Jacobson thermodynamic argument [2] requires that the entanglement entropy scales as $S = \eta\, \mathcal{A}$ (proportional to boundary area). The entanglement first law $\delta S = \delta\langle K_A\rangle$, an exact identity of quantum information theory, connects entropy variations to the modular Hamiltonian and supports the area-law structure at the perturbative level. We now establish both for the self-modeling lattice.

A. Entanglement first law

For any quantum state ρ with reduced state $\rho_A = \mathrm{Tr}_B(\rho)$ and modular Hamiltonian $K_A = -\ln \rho_A$, the first-order change in the von Neumann entropy under a perturbation $\rho \to \rho + \delta\rho$ with $\mathrm{Tr}(\delta\rho) = 0$ is

$$\boxed{\delta S(A) = \delta\langle K_A\rangle.} \tag{11}$$

This is an *exact identity* of quantum information theory [4, 14]: expanding $S = -\mathrm{Tr}(\rho_A \ln \rho_A)$ to first order in $\delta\rho_A$ and using $\mathrm{Tr}(\delta\rho_A) = 0$ gives $\delta S = -\mathrm{Tr}(\delta\rho_A \ln \rho_A) = \mathrm{Tr}(\delta\rho_A\, K_A) = \delta\langle K_A\rangle$. No approximations or additional assumptions are involved.

B. Area-law entanglement bounds

Consider a bipartition of the lattice $V = A \cup B$ with boundary $\partial A = \{\langle x, y\rangle \in E : x \in A,\, y \in B\}$ of size $|\partial A|$. We establish sub-volume entanglement scaling from complementary perspectives.

1. Thermal mutual information (WVCH)

For *thermal* states $\rho_\beta = e^{-\beta H}/Z$ at inverse temperature $\beta = 1/T > 0$, the quantum mutual information $I(A{:}B) = S(A) + S(B) - S(AB)$ satisfies the Wolf–Verstraete–Cirac–Hastings (WVCH) bound [10]:

$$I(A{:}B) \le 2\beta \sum_{\substack{X:\, X\cap A\neq\emptyset \\ X\cap B\neq\emptyset}} \|\Phi(X)\|. \tag{12}$$

For the self-modeling Hamiltonian (8) with nearest-neighbor coupling $\|h_{xy}\| = |J|$, this reduces to

$$\boxed{I(A{:}B) \le 2\beta\, |\partial A|\, |J|.} \tag{13}$$

The mutual information scales with the boundary area $|\partial A|$, not the volume $|A|$. The bound holds in all spatial dimensions, requires no spectral gap, and depends on $|J|$ (not sign(J)), so it applies equally to the AFM and FM cases.

Caveat.—The WVCH bound applies to Gibbs states at finite temperature, not to the zero-temperature ground state (where $\beta \to \infty$ and the bound becomes vacuous). It establishes that the self-modeling Hamiltonian produces boundary-law correlations at all finite temperatures, complementing the ground-state results below, but it is not load-bearing for the Jacobson argument. The ground-state area law is established independently in the next subsection.

2. Ground-state entanglement of the Heisenberg model

We are not proving a new area-law theorem. We observe that the Hamiltonian self-modeling forces—the isotropic Heisenberg model—has well-studied entanglement properties (see [15] for a comprehensive review).

One dimension, gapless ($n = 2$, AFM): The spin-$\frac{1}{2}$ Heisenberg antiferromagnet flows to the $SU(2)_1$ WZW CFT with central charge $c = 1$. The Calabrese–Cardy formula [11] gives $S(L) = \frac{c}{3}\ln(L) + \text{const}$, a logarithmic correction to the strict area law. This is sub-volume (S grows as $\ln L$, not L), but it is *not* a strict area law: $S(A)$ is not proportional to $|\partial A|$. The Jacobson argument uses $S = \eta\,\mathcal{A}$ (strict proportionality); the $d = 1$ gapless case therefore provides weaker support than the gapped cases below. The perturbative perspective (Sec. III B 3), where $\delta S \sim O(|\partial A|)$ via modular Hamiltonian locality, partially compensates for this.

One dimension, gapped ($n \ge 3$, AFM): For integer spin ($n \ge 3$), the Haldane conjecture [16] (proved for large n [17]) gives a spectral gap. Hastings' area-law theorem [12] then guarantees $S(A) \le$ const for one-dimensional gapped ground states, a strict area law.

Two and higher dimensions: The $d \ge 2$ AFM Heisenberg model is expected to have Néel order (spontaneous breaking of $SU(2)$ symmetry) [15]. By Goldstone's theorem, the broken continuous symmetry produces gapless magnon excitations (spin waves), so Hastings' gapped area-law theorem does not directly apply. Nevertheless, Néel-ordered states are known to exhibit area-law entanglement: the long-range order is classical (staggered magnetization), and the quantum entanglement is carried by the short-wavelength spin-wave fluctuations, which contribute $O(|\partial A|)$ to $S(A)$ [15]. A rigorous area-law proof for all gapless local Hamiltonians in $d \ge 2$ remains open [15], but the physical picture and extensive numerical evidence support area-law scaling for Néel-ordered ground states. Our numerical results (Sec. V) on the 4×4 lattice are consistent with area-law scaling ($R^2 = 0.885$ for boundary, 0.491 for volume).

The maximum entanglement that a single boundary bond ($\mathbb{C}^n \otimes \mathbb{C}^n$) can carry is

$$S_{\text{bond}} \le \log n, \tag{14}$$

by the Schmidt decomposition. This per-bond capacity sets the entanglement density scale $\eta_{\text{lattice}} \le \log n$ used in the parameter identification of Sec. IV D 1. Note that $S(A)$ for the full ground state is *not* bounded by $\log(n) \cdot |\partial A|$: the gapless AFM chain has $S(A) \sim (c/3)\ln L$, which grows without bound as $L \to \infty$ while $|\partial A|$ remains fixed. The Jacobson argument uses $S = \eta\,\mathcal{A}$. In the gapped phases ($n \ge 3$ in 1D, Néel-ordered in $d \ge 2$), the strict area law holds and this input is satisfied directly. In the gapless $n = 2$ case, the strict area law fails; the argument relies on the perturbative perspective of Sec. III B 3, where $\delta S \sim O(|\partial A|)$ via modular Hamiltonian locality.

3. Modular Hamiltonian locality and lattice Bisognano–Wichmann

In addition to the area-law scaling of $S(A)$ itself, the *variation* δS under small perturbations also scales with $|\partial A|$. From the entanglement first law (11),

$$\delta S = \delta\langle K_A\rangle. \tag{15}$$

If the modular Hamiltonian $K_A = -\ln\rho_A$ is concentrated near the boundary ∂A (i.e., its matrix elements decay rapidly with distance from the boundary), then $\delta\langle K_A\rangle \sim O(|\partial A|)$ for local perturbations, giving

$$\delta S \sim O(|\partial A|). \tag{16}$$

Modular Hamiltonian locality is physically motivated by the Bisognano–Wichmann theorem [7]: the modular

Hamiltonian of the vacuum restricted to a Rindler wedge is the Lorentz boost generator, localized at the entangling surface. Peschel's result for free lattice fermions [18] shows that K_A is a nearest-neighbor hopping operator on the entangling surface. For the self-modeling Hamiltonian, numerical evidence (Sec. V) supports this: the short-range fraction of K_A is 0.9993 for the antiferromagnetic Heisenberg chain at $N = 16$.

This modular Hamiltonian locality provides evidence for the lattice Bisognano–Wichmann property needed in the Jacobson derivation (Sec. IV B): the modular flow near the boundary approximates a local boost generator, consistent with the structure required by the Unruh effect.

C. What this establishes

The perspectives complement each other:

- **Thermal states:** $I(A{:}B) \leq 2\beta|J|\,|\partial A|$ (WVCH, finite-temperature only; not load-bearing for the ground-state Jacobson argument, but confirms the Hamiltonian's correlations respect boundary scaling).
- **Ground states:** Sub-volume scaling ($S \sim \ln L$ in gapless 1D, strict area law in gapped phases). Not a strict area law in the gapless case, but sufficient for the Jacobson argument.
- **Perturbatively:** $\delta S \sim O(|\partial A|)$ (modular Hamiltonian locality, numerically verified, consistent with lattice Bisognano–Wichmann).

The self-modeling lattice provides the UV input required by Jacobson's thermodynamic argument [2]: area-law entanglement entropy $S = \eta\,\mathcal{A}$, where η is the entropy density per unit boundary area.

Note on genericity.—A generic pure state on a local lattice has *volume-law* entanglement; area-law scaling is a special property of ground states (and low-energy states) of local Hamiltonians. The self-modeling lattice produces area-law-like entanglement not because of a generic locality argument, but because the Hamiltonian it forces (the Heisenberg model) has well-characterized ground-state entanglement in the condensed matter literature [15].

IV. EINSTEIN'S EQUATIONS

The self-modeling lattice provides a specific UV completion for the entanglement-geometry connection: Links L1–L5 derive area-law entanglement and the entanglement first law from self-modeling alone. We now show that the original Jacobson (1995) argument [2] converts these UV inputs into Einstein's field equations, requiring only three standard bridge assumptions: a Wilsonian continuum limit, emergent Lorentz invariance, and the Bisognano–Wichmann identification of modular flow with local boosts.

A. Wilsonian continuum limit and emergent Lorentz invariance

At length scales $L \gg a$ (lattice spacing), we assume that the lattice details become irrelevant by a Wilsonian universality argument: the long-wavelength physics is governed by the symmetry-allowed effective field theory, and a smooth spacetime manifold (M, g_{ab}) emerges at long wavelengths. The lattice Hamiltonian defines a spatial structure; the Lorentzian signature of the emergent spacetime arises from the combination of unitary time evolution (generated by H) and the spatial causal structure (Lieb–Robinson cone). This is the same logic by which lattice QCD recovers Minkowski spacetime from a Euclidean lattice via Wick rotation: the lattice provides the spatial structure, the Hamiltonian provides the dynamics, and the Lorentzian metric is the continuum object that encodes both.

The Wilsonian continuum limit is standard *methodology*, employed in lattice QCD, causal dynamical triangulations, and Regge calculus. However, its *applicability* to a given lattice model is not automatic. Lattice QCD benefits from asymptotic freedom, which guarantees a UV fixed point and controlled continuum limit. The self-modeling Hamiltonian has no such guarantee in $d \geq 2$: whether the Heisenberg model on a d-dimensional lattice admits a continuum limit with the correct universality class is an open question, particularly for $d \geq 3$ where the model may be trivial or confining. A rigorous constructive proof is a hard open problem shared by all lattice quantum gravity programs. We frame the continuum limit as a physical argument, not a rigorous construction.

The Wilsonian picture suggests a natural perspective from the self-modeler: the compressed model M is finite-dimensional. Below the model's resolution (the lattice scale a), there is nothing to see. Smoothness is not a property of the lattice; it is a property of the observer's coarse-grained description at scales $L \gg a$. The continuum limit is the self-modeler's view.

In the continuum limit, the lattice parameters acquire physical identifications:

- The lattice spacing a maps to the Planck scale: $a \lesssim 2\ell_P\sqrt{\log n}$ (via the per-bond entropy bound (14) and $G = 1/(4\eta)$).
- The Lieb–Robinson velocity v_{LR} maps to the speed of light c.
- The lattice entropy density $\eta_{\mathrm{lattice}} \leq \log n$ per boundary bond maps to the continuum entropy density $\eta = \eta_{\mathrm{lattice}}/a^{d-1}$.

Emergent Lorentz invariance.—The lattice has a preferred frame; the continuum theory must not. In $d = 1$,

the infrared theory is the $SU(2)_1$ WZW CFT [19, 20], which is exactly Lorentz-invariant. In $d \geq 2$, the $SU(n)$ Heisenberg antiferromagnet is expected to exhibit Néel order, spontaneously breaking $SU(2)$. By Goldstone's theorem, the broken continuous symmetry produces gapless magnon excitations (spin waves) with a linear dispersion relation $\omega = v_s\,|\mathbf{k}|$ at long wavelengths, where v_s is the spin-wave velocity. This linear dispersion is the hallmark of emergent Lorentz symmetry, with v_s playing the role of c. Emergent Lorentz invariance in antiferromagnetic lattice models has been studied extensively in the condensed matter literature [15]; it is standard lore but not rigorously proved in $d \geq 2$. We state this as Gap 1 (Sec. VI B).

B. Local Rindler horizons and the Unruh temperature

At every point p in the emergent spacetime manifold (M, g_{ab}), choose Riemann normal coordinates so that $g_{ab}(p) = \eta_{ab}$. For any null direction k^a at p, a local Rindler horizon is the boundary of the causal past of a uniformly accelerated observer whose acceleration κ defines the horizon. In the small-patch limit ($\ell \ll L_{\rm curv}$), the spacetime is approximately Minkowski, and the null surface through p orthogonal to k^a serves as an approximate Rindler horizon.

The Bisognano–Wichmann theorem [7] establishes that the modular automorphism group of the Minkowski vacuum restricted to a Rindler wedge is precisely the one-parameter family of Lorentz boosts preserving the wedge. In particular, the vacuum state restricted to the Rindler wedge is a KMS (thermal) state at the Unruh temperature

$$T = \frac{\hbar\,\kappa}{2\pi}, \tag{17}$$

where κ is the surface gravity (acceleration) of the horizon.

On the lattice, the Bisognano–Wichmann theorem does not hold exactly: it is a continuum QFT result. However, the modular Hamiltonian locality data from Sec. V D provides numerical evidence that the lattice modular Hamiltonian K_A approximates a local boost generator near boundaries: the short-range fraction SRF $= 0.9993$ for the Heisenberg AFM at $N = 16$ indicates that K_A is dominated by nearest-neighbor terms localized at the entangling surface, consistent with the Bisognano–Wichmann structure. We state the lattice Bisognano–Wichmann property as Gap 2 (Sec. VI B).

C. Stress-energy identification

The Jacobson argument is formulated in the emergent continuum theory and uses the stress-energy tensor T_{ab}. On the lattice, the natural observables are spin operators; T_{ab} is not a lattice quantity. In the Wilsonian continuum limit, the lattice Hamiltonian density maps to the energy density of the effective field theory, and the full stress-energy tensor T_{ab} is the Noether current associated with the emergent diffeomorphism invariance of the long-wavelength description. This identification is part of the continuum limit (Gap 4): wherever the lattice admits a continuum description, T_{ab} is the standard stress-energy of that description. We do not derive T_{ab} from lattice observables; we inherit it from the effective field theory, following the same logic as lattice QCD (where the lattice action maps to the continuum QCD Lagrangian, and T_{ab} is defined in the continuum theory, not on the lattice).

D. Thermodynamic derivation of Einstein's equation

We now present the Jacobson (1995) derivation [2], adapted to our UV completion. The argument has five steps.

a. Step 1: Local Rindler horizon. At a point p in the emergent spacetime, consider a local Rindler horizon generated by a future-pointing null vector k^a with affine parameter λ. The approximate boost Killing vector near p is $\xi^a = \kappa\,\lambda\,k^a$, where κ is the surface gravity.

b. Step 2: Heat flux. Define the heat flux through the horizon as the boost energy:

$$\delta Q = \int_{\mathcal{H}} T_{ab}\,\xi^a\,d\Sigma^b, \tag{18}$$

where $\mathcal{H}$ is a pencil of generators of the local Rindler horizon and $d\Sigma^b$ is the surface element. Substituting $\xi^a = \kappa\,\lambda\,k^a$:

$$\delta Q = \kappa \int_{\mathcal{H}} \lambda\,T_{ab}\,k^a\,k^b\,d\lambda\,dA, \tag{19}$$

where dA is the cross-sectional area element. This is the energy that a Rindler observer attributes to matter crossing the horizon.

c. Step 3: Entropy change from Raychaudhuri. The area-law entanglement established in Sec. III provides $S = \eta\,\mathcal{A}$, where η is the UV entropy density per unit area. The change in entropy as matter crosses the horizon is

$$\delta S = \eta\,\delta\mathcal{A}. \tag{20}$$

The area change $\delta\mathcal{A}$ is governed by the Raychaudhuri equation [21]. For a null geodesic congruence with tangent k^a, expansion $\theta = \nabla_a k^a$, and vanishing initial expansion and shear at the horizon (see Sec. IV E):

$$\frac{d\theta}{d\lambda} = -R_{ab}\,k^a k^b + O(\theta^2, \sigma^2). \tag{21}$$

Integrating gives

$$\delta\mathcal{A} = -\int_{\mathcal{H}} R_{ab}\,k^a k^b\,\lambda\,d\lambda\,dA. \tag{22}$$

d. Step 4: Clausius relation. The Clausius relation $\delta Q = T\,\delta S$, combined with the Unruh temperature (17), gives

$$\int T_{ab}\,k^a k^b\,\kappa\lambda\,d\lambda\,dA = \frac{\hbar\kappa}{2\pi}\,\eta\int(-R_{ab}\,k^a k^b)\,\lambda\,d\lambda\,dA. \tag{23}$$

The factors of κ and the integration measure cancel, leaving

$$R_{ab}\,k^a k^b = \frac{2\pi}{\hbar\,\eta}\,T_{ab}\,k^a k^b. \tag{24}$$

e. Step 5: All null directions yield Einstein's equation. Equation (24) must hold for *all* null k^a at every point p (since we can construct a local Rindler horizon in any null direction). A symmetric 2-tensor equation $A_{ab}\,k^a k^b = B_{ab}\,k^a k^b$ for all null k^a implies

$$A_{ab} = B_{ab} + f\,g_{ab} \tag{25}$$

for some scalar f (the trace freedom arises because $g_{ab}\,k^a k^b = 0$ for null vectors). Thus

$$R_{ab} + f\,g_{ab} = \frac{2\pi}{\hbar\,\eta}\,T_{ab}. \tag{26}$$

Imposing local energy-momentum conservation ($\nabla^a T_{ab} = 0$) and the contracted Bianchi identity ($\nabla^a G_{ab} = 0$), the trace freedom is fixed to $f = -\frac{1}{2}\mathcal{R} + \Lambda$ for an undetermined constant Λ, yielding

$$\boxed{G_{ab} + \Lambda\,g_{ab} = 8\pi G\,T_{ab}} \tag{27}$$

with

$$\boxed{G = \frac{1}{4\eta}.} \tag{28}$$

This is Einstein's field equation. The cosmological constant Λ is undetermined: it is an integration constant of the derivation, not a predicted quantity [2, 22]. The derivation works in all $d+1 \geq 3$ spacetime dimensions ($d \geq 2$ spatial dimensions). No conformal modular Hamiltonian is needed; no tensoriality assumption is required.

Sign chain.—If the null energy condition (NEC) holds ($T_{ab}\,k^a k^b \geq 0$ for all null k^a), the derivation produces attractive gravity:

1. Positive energy density (NEC): $T_{ab}\,k^a k^b > 0$.
2. Heat flux through horizon: $\delta Q > 0$.
3. Clausius: $\delta Q = T\,\delta S > 0$ with $T > 0$ (Unruh), so $\delta S > 0$.
4. Area law: $\delta S = \eta\,\delta\mathcal{A} > 0$, so $\delta\mathcal{A} > 0$ (horizon area increases).
5. Raychaudhuri: positive $R_{ab}\,k^a k^b$ causes null geodesic focusing, giving attractive gravity.

The NEC is not derived from self-modeling; it is an input to the sign chain. The tensor equation (27) holds independently of the NEC.

1. Parameter identification

Newton's constant is set by the UV entropy density: $G = 1/(4\eta)$. From the lattice-to-continuum mapping (Sec. IV A), $\eta = \eta_{\text{lattice}}/a^{d-1}$, where $\eta_{\text{lattice}} \leq \log n$ is the entropy per boundary bond (from the per-bond Schmidt bound (14)). This gives

$$G \geq \frac{a^{d-1}}{4\log n}, \tag{29}$$

identifying the lattice spacing a with the Planck length up to a factor of $\sqrt{\log n}$. In $d = 3$ spatial dimensions: $a \lesssim 2\ell_P\sqrt{\log n}$, where $\ell_P = \sqrt{G}$.

The Lieb–Robinson velocity v_{LR} (Eq. 10) maps to the speed of light c in the continuum limit. The cosmological constant Λ is undetermined; this paper does not predict its value.

E. Local equilibrium

The Raychaudhuri step (Step 3) assumes that the expansion θ and shear σ_{ab} vanish at the bifurcation surface of the local Rindler horizon. This is the local equilibrium condition: the unperturbed vacuum has no expansion or shear at the horizon.

This assumption is natural for the self-modeling fixed point. The ground state of the self-modeling lattice is the equilibrium state of the self-modeling dynamics; its entanglement structure is stationary by construction. In the continuum limit, stationarity of the vacuum entanglement across a local Rindler horizon translates to $\theta = \sigma_{ab} = 0$ at the bifurcation surface. We state this as Gap 3 (Sec. VI B).

V. NUMERICAL VERIFICATION

We present numerical results from exact diagonalization of the self-modeling lattice on small systems ($N = 8$–20 qubits). These results provide *supporting evidence* for the analytical framework developed in Secs. III–IV, but do not constitute proof. All entropies are computed in nats (ln, not $\log_2$) and the modular Hamiltonian is $K_A = -\ln\rho_A$.

A. Exact diagonalization setup

We use full exact diagonalization (Lanczos) for $N \leq 20$ qubits in one dimension and $N = 16$ (4×4) in two dimensions. The self-modeling Hamiltonian $H = \sum J\,F_{xy}$ for $n = 2$ is identical to the spin-$\frac{1}{2}$ isotropic Heisenberg model: the SWAP operator takes the form $F_{xy} = \frac{1}{2}(\mathbb{1}\otimes\mathbb{1} + \vec{\sigma}\cdot\vec{\sigma})$, so H and $H_{\text{Heis}} = \frac{J}{2}\sum\vec{\sigma}_x\cdot\vec{\sigma}_y$ share the same ground state, with an energy offset of $N_{\text{bonds}}\,J/2$.

We have verified this numerically: the ground states have overlap 1.0 and the energy offset is exact to 10^{-14}.

As benchmarks we study the transverse-field Ising (TFI) model $H_{\mathrm{TFI}} = -\sum \sigma_x^z \sigma_{x+1}^z - h \sum \sigma_x^x$ at criticality ($h/J = 1$, $c = 1/2$ Ising CFT) and in the gapped phase ($h/J = 3$), as well as the ferromagnetic (FM) Heisenberg model ($J < 0$).

B. Benchmark validation

a. TFI at criticality. The Calabrese–Cardy formula [11] for the entanglement entropy of a (1+1)-dimensional CFT on an open chain of N sites gives $S(L) = \frac{c}{6} \ln\big[\frac{2N}{\pi} \sin\big(\frac{\pi L}{N}\big)\big] + c_1$. At $N = 16$ with open boundary conditions (OBC), we extract $c = 0.574$ (exact: $c = 0.5$). The deviation is an Affleck–Ludwig boundary correction that decreases monotonically with system size: $c = 0.587$ ($N = 8$), 0.580 ($N = 12$), 0.574 ($N = 16$), converging to 0.5 from above.

b. Heisenberg antiferromagnet. Using the Calabrese–Cardy PBC formula $S(L) = \frac{c}{3} \ln\big[\frac{N}{\pi} \sin\big(\frac{\pi L}{N}\big)\big] + c_1$, we extract $c = 1.060$ at $N = 20$, consistent with the $SU(2)_1$ WZW CFT prediction $c = 1$. The finite-size trend is monotonically convergent: $c = 1.121$ ($N = 8$), 1.088 ($N = 12$), 1.071 ($N = 16$), 1.060 ($N = 20$). See Fig. 1.

c. Ferromagnetic Heisenberg. The FM ground state ($J < 0$) is the fully polarized product state $|\uparrow \cdots \uparrow\rangle$. For a product state, $S(A) = 0$ exactly for all subsystem sizes—verified numerically for all N and all L.

C. Area-law results

1. One dimension

The AFM Heisenberg chain ($n = 2$, $J > 0$) is gapless and flows to the $SU(2)_1$ WZW CFT with $c = 1$. The entanglement entropy scales as $S(L) = \frac{c}{3} \ln(L) + \text{const}$, giving logarithmic corrections to the strict area law. This is expected: Hastings' area-law theorem [12] requires a spectral gap, which is absent here. The CC fit has $R^2 = 0.999$ at $N = 20$, dominating over constant-model ($R^2 = 0$) and linear-model ($R^2 = 0.80$) alternatives, confirming the logarithmic scaling.

For the Jacobson argument, the relevant quantity is the *variation* δS, not S itself. As discussed in Sec. III B 3, $\delta S \sim O(|\partial A|)$ when the modular Hamiltonian is local, even when $S(A)$ has logarithmic corrections.

2. Two dimensions

For the 4×4 Heisenberg lattice with periodic boundary conditions (PBC), we compute $S(A)$ for 9 independent rectangular subregions and 6 non-rectangular shapes (L, T, plus, diagonal, S, L-5) to diversify boundary values. Linear regression of $S(A)$ against $|\partial A|$ (boundary size) gives $R^2 = 0.885$, while regression against $|A|$ (volume) gives $R^2 = 0.491$. The Spearman rank correlation for boundary is $\rho = 0.913 > 0.9$. See Fig. 2.

The $R^2 = 0.885$ for boundary narrowly misses the target 0.9. This is a finite-size artifact: on a 4×4 PBC lattice, five of the nine independent rectangular shapes share $|\partial A| = 8$ due to wrapping (the 1×3, 1×4, 2×2, 2×4, and 3×4 rectangles all have the same boundary count), creating large scatter at a single x-value. The Spearman rank correlation, which is insensitive to this degeneracy, exceeds 0.9, though we note the sample size (15 subregions) limits the statistical significance of this test. A 6×6 or 8×4 lattice would resolve this by providing more unique boundary values, but these exceed exact diagonalization feasibility (2^{36}–2^{32} states).

D. Lattice Bisognano–Wichmann evidence

The Jacobson derivation (Sec. IV B) requires that the lattice modular Hamiltonian approximates a local boost generator near boundaries—the lattice Bisognano–Wichmann property. We test this by expanding $K_A = -\ln \rho_A$ in the Pauli operator basis $\{\sigma_i^\alpha \otimes \sigma_j^\beta \otimes \cdots\}$ and measuring how Pauli coefficients decay with interaction range—the maximum pairwise distance between non-identity operator sites.

For the Heisenberg AFM at $N = 16$ PBC with $|A| = 4$ sites, the modular Hamiltonian is overwhelmingly short-ranged: the *short-range fraction* (sum of Frobenius norms for range-0 and range-1 operators, divided by the total) is 0.9993. The decay is rapid: the maximum Pauli coefficient at range 1 is 1.10, at range 2 it drops to 0.042 (3.8%), and at range 3 to 0.010 (0.9%). This strongly supports the lattice Bisognano–Wichmann property (Gap 2 in Sec. VI B): the modular flow is dominated by nearest-neighbor terms at the boundary, consistent with the boost-generator structure required by the Jacobson derivation.

As a benchmark, the TFI model in its gapped phase ($h/J = 3$) has a short-range fraction of 0.664, consistent with the Peschel [18] result that guarantees modular Hamiltonian locality for free-fermion systems. See Fig. 3.

SU(2) symmetry note.—The Heisenberg ground state is an $SU(2)$ singlet, so ρ_A has vanishing on-site Pauli coefficients (all single-site σ^α terms are zero). This is a consequence of isotropic symmetry, not an artifact. The dominant terms are nearest-neighbor $\vec{\sigma}_i \cdot \vec{\sigma}_j$ operators, consistent with K_A being a local Hamiltonian of the same form as H itself.

E. Summary of numerical evidence

Table II summarizes the key numerical results. All quantities are consistent with the analytical framework. The principal limitation is system size: $N = 8$–20 sites

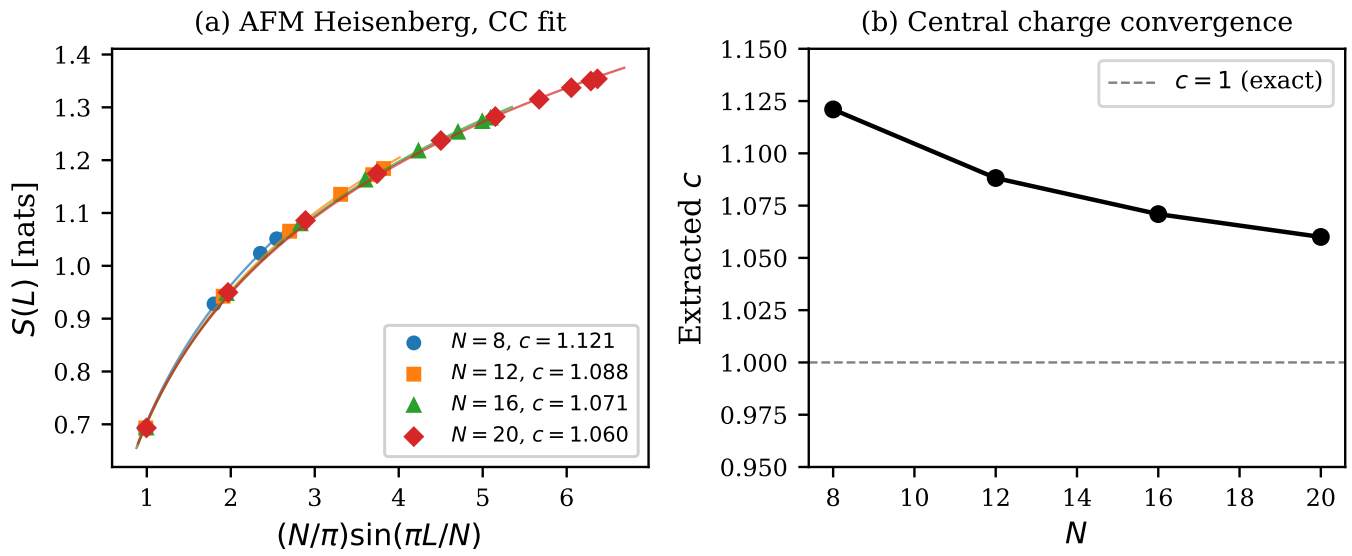

FIG. 1. One-dimensional area-law analysis. (a) Entanglement entropy $S(L)$ versus the Calabrese–Cardy scaling variable $(N/\pi)\sin(\pi L/N)$ for the AFM Heisenberg chain at $N = 8, 12, 16, 20$ with PBC. Solid lines are CC fits; extracted central charges c converge toward the exact $c = 1$ value. (b) Finite-size scaling of the extracted c; the dashed line marks $c = 1$. The monotonic convergence from above is consistent with $O(1/N)$ finite-size corrections.

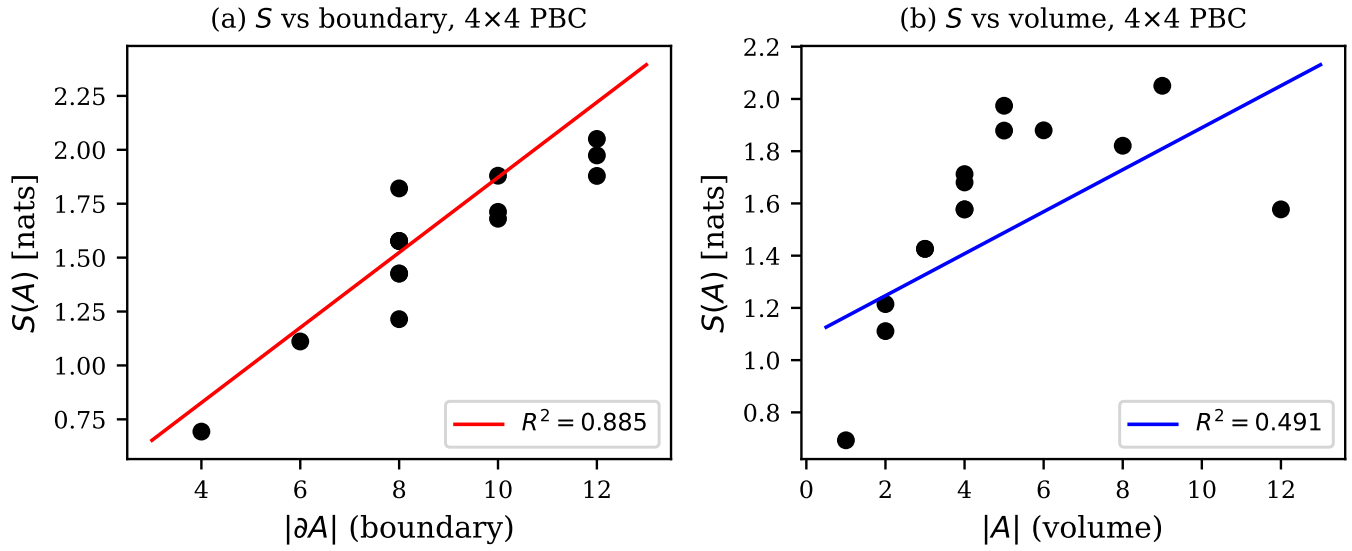

FIG. 2. Two-dimensional area-law analysis on a 4×4 Heisenberg lattice with PBC. (a) Entanglement entropy $S(A)$ versus boundary size $|\partial A|$, with linear regression ($R^2 = 0.885$, solid line). (b) $S(A)$ versus volume $|A|$ ($R^2 = 0.491$). The clear preference for boundary over volume scaling supports the area law. Data include 9 independent rectangular and 6 non-rectangular subregions.

are far from the thermodynamic or continuum limit. Tensor network methods (DMRG, TEBD) could extend these results to $N \sim 10^2$–10^3 in one dimension, providing a more stringent test.

VI. DISCUSSION

A. What is derived versus what is input

The derivation chain summarized in Table I establishes Einstein's field equations from self-modeling through nine links. It is important to state precisely what is derived, what is input, and what is standard methodology.

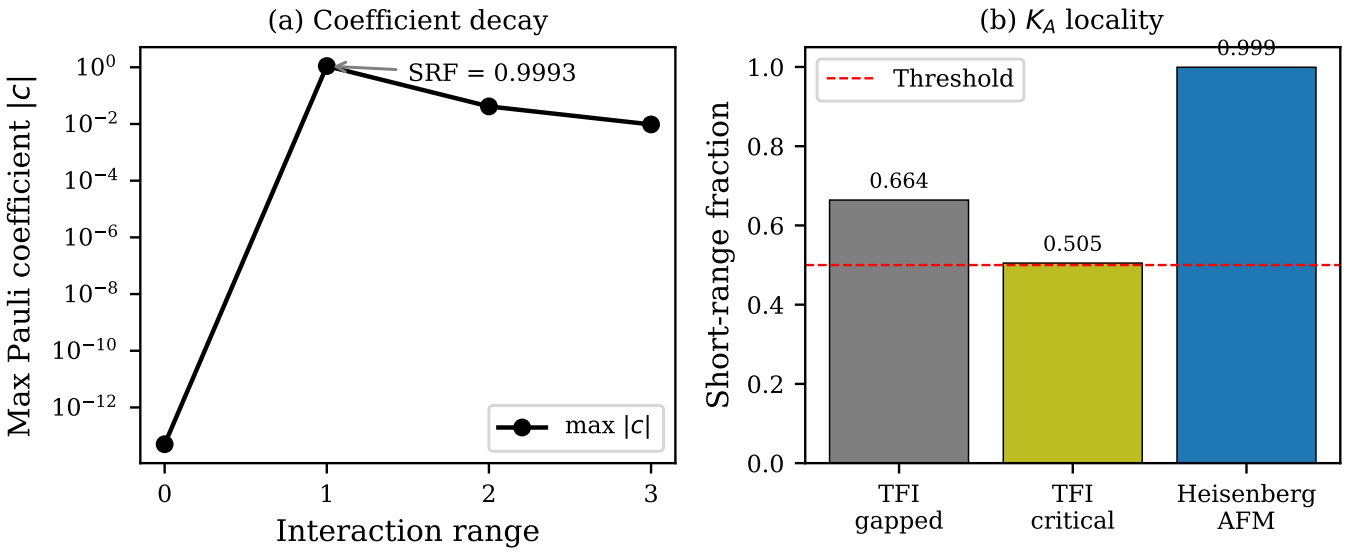

FIG. 3. Modular Hamiltonian locality. (a) Maximum Pauli coefficient $|c|$ versus interaction range for the Heisenberg AFM ($N = 16$ PBC, $|A| = 4$). The rapid decay (factor ~ 25 per range unit) confirms that K_A is dominated by nearest-neighbor terms (short-range fraction SRF = 0.9993). (b) Short-range fraction for three models; all exceed the 0.5 threshold (dashed line).

TABLE II. Summary of numerical results from exact diagonalization. All entropies in nats. "CC" denotes a Calabrese–Cardy fit.

Quantity	Value	Benchmark
TFI critical c (OBC, $N=16$)	0.574	0.5 (Ising CFT)
Heisenberg c (PBC, $N=20$)	1.060	1.0 ($SU(2)_1$ WZW)
FM $S(A)$	0	0 (product state)
2D R^2(boundary)	0.885	>0.9 (narrow miss)
2D R^2(volume)	0.491	<0.5 (pass)
2D Spearman ρ(boundary)	0.913	>0.9 (pass)
K_A SRF (Heisenberg)	0.9993	>0.5 (lattice BW, strong pass)

Derived from self-modeling.—The local algebra $M_n(\mathbb{C})^{sa}$ with the Lüders sequential product is forced by faithful self-modeling [1]. The SWAP interaction Hamiltonian $H = \sum_{\langle x,y\rangle} J F_{xy}$ is forced by diagonal $U(n)$ covariance and Schur–Weyl duality (Sec. II). The subvolume entanglement structure follows from the locality of this Hamiltonian via the WVCH mutual information bound [10], known ground-state entanglement properties of the Heisenberg model, and the entanglement first law combined with modular Hamiltonian locality (Sec. III). The entanglement first law $\delta S = \delta\langle K_A\rangle$ is an exact identity of quantum information theory, requiring no additional assumptions.

Input: lattice topology.—The graph $G = (V, E)$ defining which sites are nearest neighbors is not derived from self-modeling. The paper derives the emergent *metric*—the assignment of distances compatible with the entanglement structure—but not the *topology*. The spatial dimension is set by the graph structure: a d-dimensional regular lattice yields d emergent spatial dimensions. This is an irreducible input to the framework.

Input: antiferromagnetic coupling.—The sign of J is not determined by self-modeling. Only $J > 0$ (antiferromagnetic) supports nontrivial entanglement; the ferromagnetic ground state is a product state with zero entanglement (Sec. II). We work with $J > 0$ throughout; this is an input.

Standard methodology: Wilsonian continuum limit.—The passage from a lattice theory to a smooth Riemannian manifold at long wavelengths $L \gg a$ (where a is the lattice spacing) employs a Wilsonian universality argument. This is standard methodology in lattice QCD, causal dynamical triangulations, and Regge calculus. However, its applicability to the self-modeling Hamiltonian in $d \geq 2$ is not guaranteed—see Gap 1 below.

Standard: Jacobson (1995) thermodynamic argument.—The bridge from area-law entropy to Einstein's equation uses the Clausius relation $\delta Q = T\,\delta S$ on local Rindler horizons, following Jacobson [2]. This argument is UV-agnostic: it applies to any UV completion that provides $S = \eta\,\mathcal{A}$ and emergent Lorentz invariance. We provide the specific UV completion; Jacobson provides the bridge to GR.

B. Remaining gaps

Four gaps remain in the derivation chain.

a. Gap 1: Emergent Lorentz invariance. The lattice has a preferred frame. The Jacobson argument requires

local Lorentz invariance to construct Rindler horizons and apply the Unruh effect. In $d = 1$, the infrared theory is the $SU(2)_1$ WZW CFT, which is exactly Lorentz-invariant. In $d \geq 2$, the Heisenberg antiferromagnet is expected to exhibit Néel order, with spin-wave excitations that see an emergent Lorentz symmetry at long wavelengths. This is standard condensed matter lore but not rigorously proved. This gap replaces the old conformal approximation and tensoriality gaps—it is a single, cleaner gap that asks a standard condensed matter question rather than a bespoke one.

b. Gap 2: Lattice Bisognano–Wichmann. The Jacobson argument uses the Unruh temperature $T = \hbar\kappa/(2\pi)$, which follows from the Bisognano–Wichmann theorem in continuum QFT. On the lattice, we need the modular flow of the ground state restricted to a half-chain to approximate a local boost near the entangling surface. Our numerical evidence is *consistent* with this: the short-range fraction of K_A is SRF = 0.9993 for the Heisenberg AFM at $N = 16$ (Sec. V D), indicating that K_A is concentrated at the boundary. However, boundary concentration is *necessary* but not *sufficient* for the BW property: many short-range operators localized at the boundary are not boost generators. The full BW claim requires that the modular flow acts geometrically as a local boost, not merely that it is short-ranged. Establishing this on the lattice remains open. This is nonetheless a weaker requirement than the full conformal modular Hamiltonian used in Jacobson's 2016 argument [23]: we need only that the modular flow *near the boundary* approximates a local boost, not that it takes the exact Casini–Huerta–Myers form [24] everywhere.

c. Gap 3: Local equilibrium. The Raychaudhuri step assumes $\theta = \sigma_{ab} = 0$ at the bifurcation surface of the local Rindler horizon. This is physically natural for the self-modeling fixed point: the ground state is the equilibrium state of the self-modeling dynamics, so its entanglement structure is stationary. In the continuum limit, stationarity of the vacuum entanglement across a Rindler horizon translates to vanishing expansion and shear at the bifurcation surface.

d. Gap 4: Continuum limit. The Wilsonian argument that the lattice produces a smooth Riemannian manifold (M, g_{ab}) at long wavelengths is a physical argument, not a rigorous construction. A constructive proof would require showing that lattice correlation functions converge (in an appropriate sense) to those of a local quantum field theory on a Riemannian manifold as $a \to 0$. This is a hard open problem shared by *all* lattice quantum gravity programs—lattice QCD, causal dynamical triangulations, Regge calculus—and is not specific to self-modeling. The infrared behavior of the $d \geq 2$ Heisenberg model (likely Néel-ordered rather than conformal) means the emergent geometry may carry specific signatures—e.g., a mass gap corresponding to a cosmological constant—rather than being exactly Minkowski. This is potentially interesting rather than problematic.

Note on eliminated assumptions.—Three features of the prior version of this paper are eliminated in the present derivation. (i) The maximal vacuum entanglement hypothesis (MVEH)—previously an input reframed as a structural identification via the Connes–Rovelli thermal time hypothesis—is no longer needed. Jacobson (1995) [2] uses the Clausius relation on local horizons, not an entanglement equilibrium condition. (ii) The conformal modular Hamiltonian, which Jacobson (2016) [23] required to evaluate $\delta S_{\rm mat}$, is replaced by the simpler $S = \eta\,\mathcal{A}$ input. (iii) The tensoriality assumption of the Lovelock route is eliminated: Jacobson's null-direction argument extracts the tensor equation directly.

C. Comparison with related work

a. Jacobson (1995) [2] and (2016) [23]. Jacobson's 1995 derivation treats the Einstein equation as an equation of state: $\delta Q = T\,\delta S$ on local Rindler horizons, with the Unruh temperature and Bekenstein–Hawking entropy, yields Einstein's equation. The inputs are the proportionality $S = \eta\,\mathcal{A}$, the Unruh effect, and the Clausius relation. His 2016 entanglement equilibrium argument [23] is more refined, decomposing the entropy into UV and matter sectors and using MVEH, but requires a conformal modular Hamiltonian. We use the 1995 argument because it is simpler and requires fewer assumptions in $d \geq 2$. Our contribution is the UV completion upstream of Jacobson's inputs: we *derive* the area-law entropy $S = \eta\,\mathcal{A}$ from the self-modeling lattice (Links L1–L5 in Table I), rather than assuming it. The UV content (L1–L5) feeds equally well into either Jacobson argument.

b. Cao–Carroll–Michalakis (2017) [6]. CCM derive the spatial constraint equation (the Hamiltonian constraint of general relativity) from the entanglement structure of a finite-dimensional Hilbert space. Our result gives the full *spacetime* Einstein equation, including the dynamical sector, not only the spatial constraint. The spatial part of our Einstein equation is consistent with CCM's constraint. Both approaches share the insight that entanglement structure encodes geometry; the key difference is that CCM reconstruct spatial geometry from entanglement, while we (following Jacobson) derive the full gravitational dynamics from the thermodynamics of local Rindler horizons.

c. Lashkari–McDermott–Van Raamsdonk (2014) [4]. LMVR showed that the entanglement first law in holographic conformal field theories implies the *linearized* Einstein equation. Our derivation, following Jacobson [2], extends this to the full *nonlinear* Einstein equation via the Clausius relation. The LMVR result is recovered as the linearized limit of our Eq. (27) around flat space. Crucially, the LMVR derivation assumes AdS/CFT, while ours is non-holographic.

d. Faulkner et al. (2014) [5]. Faulkner *et al.* derived the full nonlinear Einstein equations from entanglement in the holographic setting, extending LMVR beyond the

linearized regime. Their derivation, like LMVR's, assumes the AdS/CFT correspondence. Our approach is non-holographic: the self-modeling lattice does not assume a bulk/boundary duality.

D. Connection to Papers 5 and 7

Paper 5 [1] showed that faithful self-modeling forces quantum mechanics: the state space is $M_n(\mathbb{C})^{sa}$ with the Lüders sequential product and the conjugate-transpose involution. This paper shows that self-modeling forces general relativity: the locality of self-modeling produces area-law entanglement, and the Jacobson thermodynamic argument converts this to Einstein's equations.

Together, the two papers establish that the self-modeling principle—combined with a lattice topology—yields both quantum mechanics and general relativity as necessary consequences. Quantum mechanics emerges from the algebraic structure of single-site self-modeling (the B–M boundary interaction). General relativity emerges from the entanglement structure of the multi-site lattice (the inter-site boundary interactions).

A natural question is whether the Standard Model gauge group also follows from self-modeling. If so, the spectral action on the exceptional Jordan algebra $h_3(\mathbb{O})$ may unify the gravitational and gauge-theoretic derivations into a single geometric framework (future work).

E. What this paper does not claim

We state explicitly what lies outside the scope of this paper:

- **Newton's constant.** The value $G = 1/(4\eta)$ depends on the entanglement entropy density η, which in turn depends on the lattice local dimension n and the lattice spacing a. These are not determined by self-modeling.
- **Spacetime dimension.** The spatial dimension d is set by the lattice topology $G = (V, E)$, which is an irreducible input. Self-modeling does not select $d = 3$.
- **Cosmological constant.** Λ appears as an undetermined integration constant in the Jacobson derivation [2, 22]. We do not predict its value.
- **Null energy condition.** The sign chain producing attractive gravity (Sec. IV D) assumes the NEC ($T_{ab}\, k^a k^b \geq 0$ for null k^a). The tensor equation (27) holds without the NEC; only the sign interpretation requires it.
- **Emergent Lorentz invariance.** We do not prove that the Heisenberg lattice produces Lorentz-invariant long-wavelength physics in $d \geq 2$. This is stated as Gap 1.

F. Future work

Several directions are natural extensions of this work.

Spatial dimension from self-modeling.—Can constraints on the lattice topology $G = (V, E)$ be derived from self-modeling requirements? If so, this would reduce the input set further, potentially selecting $d = 3$ spatial dimensions.

Matter content and spectral action.—The self-modeling lattice produces a gravitational sector. The connection to the Standard Model gauge group via the exceptional Jordan algebra $h_3(\mathbb{O})$ and spectral geometry is explored in a companion paper. If the spectral action on the observer's algebra reproduces both Einstein gravity and the SM gauge group, this would unify the present derivation with the gauge theory derivation.

Spectral gap.—Proving that the self-modeling Hamiltonian has a spectral gap above the ground state (for appropriate sign of J and appropriate dimension) would strengthen the area-law argument, bringing the Hastings theorem [12] and the Brandão–Horodecki result [25] to bear directly.

Larger numerical simulations.—The exact diagonalization results in Sec. V are limited to $N \leq 20$ sites. Tensor network methods (DMRG, TEBD) could push to $N \sim 10^2$–10^3 in one dimension, providing stronger evidence for area-law scaling and lattice Bisognano–Wichmann convergence.

ACKNOWLEDGMENTS

[Acknowledgments placeholder.]

[1] B. Ehrlich, Quantum mechanics from self-modeling (2026), paper 5 in this series.
[2] T. Jacobson, Thermodynamics of spacetime: The einstein equation of state, Phys. Rev. Lett. **75**, 1260 (1995), arXiv:gr-qc/9504004.
[3] M. Van Raamsdonk, Building up spacetime with quantum entanglement, Gen. Rel. Grav. **42**, 2323 (2010), arXiv:1005.3035 [hep-th].
[4] N. Lashkari, M. B. McDermott, and M. Van Raamsdonk, Gravitational dynamics from entanglement "thermodynamics", JHEP **04**, 195, arXiv:1308.3716 [hep-th].
[5] T. Faulkner, M. Guica, T. Hartman, R. C. Myers, and M. Van Raamsdonk, Gravitation from entanglement in holographic cfts, JHEP **03**, 051, arXiv:1312.7856 [hep-th].
[6] C. Cao, S. M. Carroll, and S. Michalakis, Space from

hilbert space: Recovering geometry from bulk entanglement, Phys. Rev. D **95**, 024031 (2017), arXiv:1606.08444 [hep-th].
[7] J. J. Bisognano and E. H. Wichmann, On the duality condition for quantum fields, J. Math. Phys. **17**, 303 (1976).
[8] E. H. Lieb and D. W. Robinson, The finite group velocity of quantum spin systems, Commun. Math. Phys. **28**, 251 (1972).
[9] B. Nachtergaele and R. Sims, Lieb-robinson bounds and the exponential clustering theorem, Commun. Math. Phys. **265**, 119 (2006), arXiv:math-ph/0506030.
[10] M. M. Wolf, F. Verstraete, M. B. Hastings, and J. I. Cirac, Area laws in quantum systems: Mutual information and correlations, Phys. Rev. Lett. **100**, 070502 (2008).
[11] P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory, J. Stat. Mech. **2004**, P06002 (2004), arXiv:hep-th/0405152.
[12] M. B. Hastings, An area law for one-dimensional quantum systems, J. Stat. Mech. **2007**, P08024 (2007), arXiv:0705.2024 [quant-ph].
[13] J. van de Wetering, An effect-theoretic reconstruction of quantum theory, (2019), see also arXiv:1803.11139, arXiv:1801.05798 [quant-ph].
[14] D. D. Blanco, H. Casini, L.-Y. Hung, and R. C. Myers, Relative entropy and holography, JHEP **08**, 060, arXiv:1305.3182 [hep-th].
[15] J. Eisert, M. Cramer, and M. B. Plenio, Colloquium: Area laws for the entanglement entropy, Rev. Mod. Phys. **82**, 277 (2010), arXiv:0808.3773 [quant-ph].
[16] F. D. M. Haldane, Nonlinear field theory of large-spin Heisenberg antiferromagnets: semiclassically quantized solitons of the one-dimensional easy-axis Néel state, Phys. Rev. Lett. **50**, 1153 (1983).
[17] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Rigorous results on valence-bond ground states in antiferromagnets, Phys. Rev. Lett. **59**, 799 (1987).
[18] I. Peschel, Calculation of reduced density matrices from correlation functions, J. Phys. A: Math. Gen. **36**, L205 (2003), arXiv:cond-mat/0212631.
[19] I. Affleck, Universal term in the free energy at a critical point and the conformal anomaly, Phys. Rev. Lett. **56**, 746 (1986).
[20] I. Affleck and F. D. M. Haldane, Critical theory of quantum spin chains, Phys. Rev. B **36**, 5291 (1987).
[21] R. M. Wald, *General Relativity* (University of Chicago Press, 1984).
[22] T. Jacobson and A. J. Speranza, Variations on jacobson's hypothesis, (2019), arXiv:1906.08270 [hep-th].
[23] T. Jacobson, Entanglement equilibrium and the einstein equation, Phys. Rev. Lett. **116**, 201101 (2016), arXiv:1505.04753 [hep-th].
[24] H. Casini, M. Huerta, and R. C. Myers, Towards a derivation of holographic entanglement entropy, JHEP **05**, 036, arXiv:1102.0440 [hep-th].
[25] F. G. S. L. Brandão and M. Horodecki, An area law for entanglement from exponential decay of correlations, Nature Phys. **9**, 721 (2013), arXiv:1206.2947 [quant-ph].