

Falsification of the Born–Fisher–Experiential Conjecture in a Qubit Toy Model

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Abstract

The experiential density $\rho_Q = I_{vN}(B; M)(1 - I_{vN}/S_{vN}(B))$ measures meaningful complexity in bipartite quantum systems by penalising both under-correlated and over-correlated regimes. The Born–Fisher–Experiential conjecture proposes that Born-rule probability assignments are dynamically selected by the half-saturation condition $I_{vN} = S_{vN}(B)/2$, where the experiential density is maximal. We test this conjecture numerically using a qubit toy model with two complementary approaches: (i) a static diagonal-state sweep over a 200×200 grid in the body-probability / tracking-accuracy parameter space, and (ii) Lindblad master equation dynamics with an exchange Hamiltonian and dephasing. The static test shows that the half-saturation tracking accuracy $\alpha_{1/2}(p)$ varies with p (spread 0.069) but Born-rule distributions display no special property compared with non-Born alternatives. The dynamical test reveals that $\rho_Q(t) \leq 0$ throughout all 1900+ Lindblad trajectories because $I_{vN}/S_{vN}(B)$ monotonically decreases from 2 (pure entangled) to 1 (perfect tracking), never entering the regime where ρ_Q is positive. The trajectory functional $\mu_Q(\theta)$ is identically zero for all initial states and all tested decoherence parameters. The conjecture is cleanly falsified for this model class. We identify the exchange Hamiltonian’s perfect-tracking behaviour as the mechanism and suggest non-Markovian channels as a potential avenue for modified conjectures.

1 Introduction

A central question in the foundations of quantum mechanics is why the Born rule holds: given a quantum state $|\psi\rangle = \sum_k c_k |k\rangle$, the probability of measuring outcome k is $p_k = |c_k|^2$. Despite its empirical success, the rule remains axiomatic in standard quantum theory. Several derivation programmes exist—Gleason’s theorem [3], operational approaches [4], and Valentini’s sub-quantum H -theorem [2]. Decision-theoretic arguments due to Deutsch [10] and Wallace [11] derive Born-rule probabilities from rationality axioms applied to Everettian branching, while Zurek’s envariance programme [12] obtains $p_k = |c_k|^2$ from entanglement symmetry. However, none of these approaches employs a bipartite mutual-information functional as a variational selector for probability assignments.

The experiential measure framework [8] introduces a scalar functional on bipartite systems that quantifies meaningful complexity. For a classical system with body B and

model M , the experiential density is

$$\rho = I(B; M) \left(1 - \frac{I(B; M)}{H(B)} \right), \quad (1)$$

where $I(B; M)$ is the mutual information and $H(B)$ is the marginal entropy. This functional vanishes at both extremes of correlation ($I = 0$: no model, $I = H$: perfect model) and peaks at half saturation $I = H/2$, where $\rho_{\max} = H/4$. The peak describes a system whose model captures exactly half of the body’s entropy—a regime of maximal “surprise-generating capacity”.

The quantum extension [8] replaces Shannon entropies with von Neumann entropies:

Definition 1 (Quantum experiential density). *For a bipartite density matrix ρ_{BM} with reduced states $\rho_B = \text{Tr}_M(\rho_{BM})$ and $\rho_M = \text{Tr}_B(\rho_{BM})$, define*

$$\rho_Q = I_{\text{vN}}(B; M) \left(1 - \frac{I_{\text{vN}}(B; M)}{S_{\text{vN}}(B)} \right), \quad (2)$$

where $S_{\text{vN}}(\rho) = -\text{Tr}(\rho \ln \rho)$ is the von Neumann entropy and $I_{\text{vN}}(B; M) = S_{\text{vN}}(B) + S_{\text{vN}}(M) - S_{\text{vN}}(BM)$ is the quantum mutual information.

A critical constraint shapes the entire analysis. For any pure bipartite state $|\psi\rangle_{BM}$, the Schmidt decomposition gives $S_{\text{vN}}(BM) = 0$ and $S_{\text{vN}}(B) = S_{\text{vN}}(M)$, so

$$I_{\text{vN}}(B; M) = 2 S_{\text{vN}}(B), \quad \rho_Q = 2 S_{\text{vN}}(B)(1 - 2) = -2 S_{\text{vN}}(B) < 0. \quad (3)$$

The quantum experiential density is *negative* for all entangled pure states because quantum mutual information can exceed the marginal entropy—a phenomenon with no classical analogue. The functional ρ_Q is therefore physically meaningful only in the decohered (mixed-state) regime where $I_{\text{vN}} \leq S_{\text{vN}}(B)$.

Conjecture 1 (Born–Fisher–Experiential). *Let $|\psi\rangle = \sum_k c_k |k\rangle$ be a quantum state measured in the pointer basis $\{|k\rangle\}$. Among all probability assignments $\{p_k\}$ over measurement outcomes, the Born-rule assignment $p_k = |c_k|^2$ is dynamically selected by the condition that the time-integrated experiential density*

$$\mu_Q(\theta) = \int_0^T \max(\rho_Q(t), 0) dt \quad (4)$$

is extremised at the Born-rule initial state, where θ parametrises the initial entanglement angle. Equivalently, the half-saturation condition $I_{\text{vN}} = S_{\text{vN}}(B)/2$ plays a distinguished role for Born-rule probability assignments.

The conjecture connects the observer-dependent experiential measure to quantum foundations: if true, it would provide a variational mechanism for why nature selects $p_k = |c_k|^2$ rather than some other probability rule. The connection to Fisher information arises through the Baez–Dolan groupoid cardinality [1], which motivates the $1/|\text{Aut}(x)|$ weighting that appears naturally in structure spaces. (The Fisher information connection is motivational; the detailed construction appears in the companion work [8] and is not computed in the present paper.)

In this paper we test conjecture 1 using a qubit toy model ($d = 2$) with two complementary numerical strategies. Test A (section 3) performs a static sweep over diagonal

body–model states, asking whether the half-saturation tracking accuracy depends on whether the body probability follows the Born rule. Test B (section 4) integrates the full Lindblad master equation starting from pure entangled states, tracking $\rho_Q(t)$ and the trajectory functional $\mu_Q(\theta)$ across $1\,900^+$ parameter combinations.

Our results are unambiguous. Test A shows that $\alpha_{1/2}(p)$ varies with p but Born-rule distributions possess no special property at half saturation. Test B shows that $\rho_Q(t) \leq 0$ throughout the entire Lindblad evolution for every tested trajectory, making $\mu_Q \equiv 0$ with no dynamical selection mechanism. The conjecture is falsified for this class of Lindblad models.

2 Model and Methods

2.1 Quantum experiential density

We work in the bipartite Hilbert space $\mathcal{H} = \mathcal{H}_B \otimes \mathcal{H}_M$ with $\dim \mathcal{H}_B = \dim \mathcal{H}_M = d$. The body subsystem B represents the physical system being observed; the model subsystem M represents the observer’s internal representation. All entropies use the natural logarithm (nats).

The quantities entering the experiential density (2) are computed from the joint density matrix ρ_{BM} via eigenvalue decomposition:

$$S_{\text{vN}}(\rho) = - \sum_i \lambda_i \ln \lambda_i, \quad (5)$$

where $\{\lambda_i\}$ are the eigenvalues of ρ , with the convention $0 \ln 0 = 0$. Eigenvalues below a cutoff $\varepsilon = 10^{-15}$ are treated as zero. Reduced density matrices are obtained by partial trace: $\rho_B = \text{Tr}_M(\rho_{BM})$, $\rho_M = \text{Tr}_B(\rho_{BM})$.

We define the *information ratio*

$$r \equiv \frac{I_{\text{vN}}(B; M)}{S_{\text{vN}}(B)}, \quad (6)$$

with the convention $r = 0$ when $S_{\text{vN}}(B) = 0$. The experiential density is positive if and only if $0 < r < 1$, i.e. the system is in the under-correlated regime. For $r > 1$ (entangled/over-correlated), $\rho_Q < 0$. For pure states, $r = 2$ always.

2.2 Diagonal BM state parametrisation

For the static test, we construct diagonal body–model states directly in the pointer basis. A qubit body with probability p of being in state $|0\rangle$ and a model with tracking accuracy α yields the joint state:

$$\rho_{BM}(p, \alpha) = p |0\rangle\langle 0| \otimes \rho_M^{(0)} + (1 - p) |1\rangle\langle 1| \otimes \rho_M^{(1)}, \quad (7)$$

where

$$\rho_M^{(0)} = \begin{pmatrix} \alpha & 0 \\ 0 & 1 - \alpha \end{pmatrix}, \quad \rho_M^{(1)} = \begin{pmatrix} 1 - \alpha & 0 \\ 0 & \alpha \end{pmatrix}. \quad (8)$$

When $\alpha = 1$ the model perfectly tracks the body ($I_{\text{vN}} = S_{\text{vN}}(B)$, $r = 1$, $\rho_Q = 0$). When $\alpha = 1/d = 1/2$ the model carries no information ($I_{\text{vN}} = 0$, $\rho_Q = 0$). For intermediate α , the experiential density is positive.

For a given p , the half-saturation tracking accuracy $\alpha_{1/2}(p)$ is defined as the value of α at which $r = 1/2$. We extract $\alpha_{1/2}$ by bisection root-finding on $r(\alpha) - 1/2 = 0$, converging to tolerance 10^{-12} .

The Born rule enters through the parametrisation of the body probability. For a quantum state $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$, the Born rule gives $p = \cos^2\theta$. Non-Born alternatives include $p = \cos^4\theta/Z$ (squared Born rule), $p = |\cos\theta|/Z$ (absolute-value rule), and $p = 1/2$ (uniform), where Z is a normalisation constant.

2.3 Lindblad master equation

For the dynamical test, we evolve a pure entangled initial state

$$|\psi(\theta)\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle \quad (9)$$

under the Lindblad master equation

$$\frac{d\rho}{dt} = -i[H_{\text{int}}, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right). \quad (10)$$

The interaction Hamiltonian is the exchange (or swap) coupling,

$$H_{\text{int}} = g(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y), \quad (11)$$

which drives the model subsystem to track the body. The collapse operator implements dephasing in the pointer basis:

$$L = \sqrt{\gamma_D}(\sigma_z \otimes I_2), \quad (12)$$

where γ_D is the dephasing rate. This gives a decoherence time $\tau_D = 1/(2\gamma_D)$.

For the asymmetric extension, we replace the single collapse operator with two projective operators:

$$L_0 = \sqrt{\gamma_0}(|0\rangle\langle 0| \otimes I_2), \quad L_1 = \sqrt{\gamma_1}(|1\rangle\langle 1| \otimes I_2), \quad (13)$$

with $\gamma_0/\gamma_1 \neq 1$ breaking the symmetry between pointer states.

2.4 Superoperator construction and integration

We vectorise the density matrix by column-stacking, $\text{vec}(\rho) = \rho.\text{flatten}(\text{'F'})$, so that $\text{vec}(A\rho B) = (B^T \otimes A) \text{vec}(\rho)$. The Lindblad equation becomes

$$\frac{d}{dt} \text{vec}(\rho) = \mathcal{L} \cdot \text{vec}(\rho), \quad (14)$$

where the superoperator \mathcal{L} is a $d^4 \times d^4$ matrix (16×16 for qubits):

$$\mathcal{L} = -i(I \otimes H - H^T \otimes I) + \sum_k \left(L_k^* \otimes L_k - \frac{1}{2} I \otimes L_k^\dagger L_k - \frac{1}{2} (L_k^\dagger L_k)^T \otimes I \right). \quad (15)$$

We integrate (14) using a fixed-step fourth-order Runge–Kutta (RK4) scheme with $n = 2000$ steps over the interval $[0, 10/\gamma_D]$. Convergence is verified by comparing with $n = 4000$ steps; the difference in μ_Q is below 10^{-15} nats·time.

At each stored time step, we reconstruct the density matrix, enforce Hermiticity by averaging $\rho \rightarrow (\rho + \rho^\dagger)/2$, compute $S_{\text{vN}}(B)$, $I_{\text{vN}}(B; M)$, the ratio r , and ρ_Q . The trajectory functional is

$$\mu_Q(\theta) = \int_0^{T_{\text{final}}} \max(\rho_Q(t), 0) dt, \quad (16)$$

evaluated by the trapezoidal rule on the stored time grid.

3 Test A: Static Diagonal States

3.1 Grid sweep

We evaluate the information ratio $r = I_{\text{vN}}/S_{\text{vN}}(B)$ and the experiential density ρ_Q on a 200×200 grid over $(p, \alpha) \in [0.01, 0.99] \times [0.51, 0.999]$ for $d = 2$. The ratio r spans $[1.4 \times 10^{-4}, 0.989]$ across the grid, confirming that the diagonal-state parametrisation covers the full under-correlated regime. The experiential density spans $[7.9 \times 10^{-6}, 0.173]$ nats, with maximum value $S_{\text{vN}}(B)/4 = \ln(2)/4 = 0.173$ nats at $p = 0.5$, $\alpha \approx 0.890$, confirming the parabolic peak structure to 0.002%.

3.2 Half-saturation curve

For each value of p in the grid, we extract $\alpha_{1/2}(p)$ by bisection on $r(\alpha) - 1/2 = 0$. The resulting curve is *not* constant: $\alpha_{1/2}$ ranges from 0.890 (at $p = 0.5$) to 0.959 (near $p \rightarrow 0$ or $p \rightarrow 1$), with a spread of 0.069 and standard deviation 0.016. The curve is symmetric about $p = 0.5$, as expected from the symmetry $I(p, \alpha) = I(1 - p, \alpha)$.

Representative values:

p	0.10	0.20	0.30	0.50	0.80	0.90
$\alpha_{1/2}$	0.916	0.901	0.894	0.890	0.901	0.916

3.3 Born vs. non-Born comparison

We parametrise the body probability p as a function of the quantum state angle $\theta \in (0, \pi/2)$ using four rules:

- (i) Born rule: $p = \cos^2 \theta$;
- (ii) Squared Born: $p = \cos^4 \theta / Z$;
- (iii) Absolute-value: $p = |\cos \theta| / Z$;
- (iv) Uniform: $p = 0.5$.

For each rule, we compute $\alpha_{1/2}(p(\theta))$ as θ varies.

The results show no distinguishing property of the Born rule. The Born-rule $\alpha_{1/2}$ curve has mean 0.910 and spread 0.069; the squared-Born curve has mean 0.916 and spread 0.069; the absolute-value curve has mean 0.898 and spread 0.041; the uniform rule trivially gives a single point at $\alpha_{1/2} = 0.890$. No rule produces a constant $\alpha_{1/2}$, no rule produces an extremum that other rules lack, and the Born rule is not distinguished in any measurable way.

3.4 Qutrit confirmation

We repeat the analysis for $d = 3$ (qutrits) with body probabilities $(p, (1-p)/2, (1-p)/2)$ and model tracking α on the diagonal. The $d = 3$ half-saturation curve also varies with p , with mean $\alpha_{1/2} = 0.848$ and spread 0.023. The qualitative behaviour matches $d = 2$: no special Born-rule property is observed. The result is not a $d = 2$ artefact.

Test A verdict: The static half-saturation conjecture is falsified. The tracking accuracy $\alpha_{1/2}(p)$ varies with p , but Born-rule body probabilities show no distinguished behaviour at half saturation.

4 Test B: Lindblad Dynamics

4.1 Pilot trajectory

Before the full parameter sweep, we examine a single trajectory with $\theta = \pi/4$ (maximally entangled), $\gamma_D = 1.0$, $g = 1.0$. At $t = 0$, the state is pure with $I_{vN} = 2 \ln 2 = 1.386$ nats and $S_{vN}(B) = \ln 2 = 0.693$ nats, giving $r = 2$ and $\rho_Q = -1.386$ nats.

As the system evolves, $S_{vN}(BM)$ increases (the state becomes mixed), reducing I_{vN} . Meanwhile $S_{vN}(B)$ remains approximately constant at $\ln 2$ (the Bell state has a maximally mixed reduced state, and local dephasing does not change the body's marginal entropy). The ratio r decreases monotonically from 2 toward 1.

Crucially, r does *not* pass through $r = 1/2$. It decreases from 2 to 1 and stops, because the exchange Hamiltonian drives the model to perfectly track the body: $I_{vN} \rightarrow S_{vN}(B)$ as $t \rightarrow \infty$. Throughout the entire trajectory, $r \geq 1$, so $\rho_Q \leq 0$ at all times. The trajectory functional $\mu_Q = 0$ identically.

4.2 Symmetric sweep: 1 200 trajectories

We sweep over 100 values of $\theta \in (0.05, \pi/2 - 0.05)$, 3 dephasing rates $\gamma_D \in \{0.1, 1.0, 10.0\}$, and 4 coupling strengths $g \in \{0.1, 0.5, 1.0, 2.0\}$, yielding 1 200 Lindblad trajectories. For every trajectory:

- $I_{vN}(t)/S_{vN}(B)(t) \geq 1.000$ at all stored time steps.
- $\rho_Q(t) \leq 0$ at all stored time steps.
- $\mu_Q(\theta) = 0$ to within 10^{-15} nats·time.

The ratio $r(t)$ decreases monotonically from 2 (pure entangled initial state) to 1 (perfect tracking steady state) for all parameter combinations. No trajectory enters the under-correlated regime $r < 1$ where $\rho_Q > 0$.

The minimum value of r across all 1 200 trajectories is 1.0000 (to numerical precision 10^{-15}). This is not a marginal result: the ratio approaches 1 from above and remains there, with no oscillation or undershoot.

4.3 Asymmetric extension: 700 trajectories

To test whether the symmetric dephasing is responsible for the trivial result, we run 700 additional trajectories with asymmetric dephasing. We fix $g = 1.0$ and $\gamma_0 + \gamma_1 = 2.0$, varying the ratio $\gamma_0/\gamma_1 \in \{0.1, 0.2, 0.5, 1.0, 2.0, 5.0, 10.0\}$ across 100 values of θ .

The result is identical: $\rho_Q(t) \leq 0$ throughout, $\mu_Q = 0$ identically for all γ_0/γ_1 ratios and all θ . Asymmetric dephasing does not create an under-correlated transient.

4.4 Qutrit spot check

Three spot-check trajectories at $d = 3$ with $\theta \in \{\pi/8, \pi/4, 3\pi/8\}$, using the generalised exchange (swap) Hamiltonian and projective dephasing, confirm the same behaviour: $r \in [1, 2]$, $\rho_Q \leq 0$, $\mu_Q = 0$. The maximum ρ_Q across the $d = 3$ trajectories is -9.6×10^{-10} nats—negative to numerical precision. The falsification is not an artefact of the qubit ($d = 2$) Hilbert space.

Test B verdict: The dynamical Born–Fisher–Experiential conjecture is falsified. The trajectory functional $\mu_Q(\theta) \equiv 0$ for all initial states and all tested decoherence parameters. No selection mechanism exists in this model class.

5 Physical Mechanism

The falsification has a clear physical origin. The exchange Hamiltonian (11) generates a “tracking” interaction that continuously swaps information from B to M . Combined with dephasing on B , the dynamics proceeds as follows.

At $t = 0$, the pure entangled state $|\psi(\theta)\rangle$ has $r = 2$: the subsystems share twice as much mutual information as the body’s marginal entropy allows classically. This is the over-correlated regime, where the excess correlation is due to entanglement. In particular, the excess mutual information $I_{\text{vN}} - S_{\text{vN}}(B)$ for these entangled states includes a quantum discord contribution [13] that has no classical analogue.

As dephasing destroys the off-diagonal coherences of ρ_{BM} , the joint entropy $S_{\text{vN}}(BM)$ increases, reducing $I_{\text{vN}} = S_{\text{vN}}(B) + S_{\text{vN}}(M) - S_{\text{vN}}(BM)$. Simultaneously, the exchange Hamiltonian rebuilds classical correlations between B and M .

The net effect is that the system transitions *directly* from the over-correlated regime ($r > 1$, entangled) to the perfectly-correlated regime ($r = 1$, $I_{\text{vN}} = S_{\text{vN}}(B)$). It never passes through an under-correlated phase ($r < 1$) where ρ_Q would be positive. The trajectory in (r, t) space is a monotone decreasing curve from $r = 2$ to $r = 1$, parameterised by the competition between dephasing (which destroys quantum correlations) and exchange (which builds classical correlations).

This mechanism is specific to the exchange-plus-dephasing model. The exchange Hamiltonian is “too good” at tracking: it ensures $I_{\text{vN}} \geq S_{\text{vN}}(B)$ at all times, preventing the system from entering the under-correlated regime. A weaker coupling mechanism or a different decoherence channel might produce temporary under-correlation ($r < 1$), which is the regime where the experiential density conjecture would have non-trivial content.

Quantitatively, the off-diagonal elements of the joint density matrix decay as $|\rho_{BM}[0, 3](t)| = |\rho_{BM}[0, 3](0)| e^{-2\gamma_D t}$ under pure dephasing (validated to relative error 5×10^{-11}), while the exchange coupling preserves the diagonal correlations. At late times $t \gg \tau_D$, the state is approximately diagonal with $I_{\text{vN}} \approx S_{\text{vN}}(B)$, giving $\rho_Q \approx 0^-$ (approaching zero from below).

The numerical observation $r(t) \geq 1$ for all trajectories admits an analytical proof for the model class considered here.

Proposition 1 (Analytical bound $r \geq 1$ for exchange-plus-dephasing). *For initial states of the form $|\psi(\theta)\rangle = \cos \theta |00\rangle + \sin \theta |11\rangle$ evolving under the Lindblad equation (10) with exchange Hamiltonian (11) and dephasing (12), the information ratio satisfies $r(t) \geq 1$ for all $t \geq 0$.*

Proof. The initial state has support only in the $\{|00\rangle, |11\rangle\}$ subspace, so the initial density matrix $\rho_{BM}(0)$ has non-zero entries only at positions $(0, 0)$, $(0, 3)$, $(3, 0)$, and $(3, 3)$ in the computational basis. We show that the diagonal populations are time-independent, which fixes the marginal entropies and forces $r \geq 1$.

Step 1: The exchange Hamiltonian preserves the populations. The exchange Hamiltonian $H = g(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y)$ satisfies $H|00\rangle = 0$ and $H|11\rangle = 0$: explicitly, $(\sigma_x \otimes \sigma_x)|00\rangle = |11\rangle$ while $(\sigma_y \otimes \sigma_y)|00\rangle = -|11\rangle$, so $H|00\rangle = g(|11\rangle - |11\rangle) = 0$, and similarly for $|11\rangle$. Therefore the commutator $[H, \rho]$ contributes only to off-diagonal coherences and does not change the diagonal entries of ρ_{BM} .

Step 2: Dephasing preserves the populations. The collapse operator $L = \sqrt{\gamma_D}(\sigma_z \otimes I)$ is diagonal in the computational basis: $L|ij\rangle = \pm\sqrt{\gamma_D}|ij\rangle$. The Lindblad dissipator $L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$ therefore preserves all diagonal matrix elements of ρ_{BM} while damping off-diagonal coherences.

Step 3: Marginal entropies are constant. Since the populations $\rho_{BM}[k, k]$ are time-independent, the partial traces $\rho_B = \text{Tr}_M(\rho_{BM})$ and $\rho_M = \text{Tr}_B(\rho_{BM})$ are constant. Hence $S_{vN}(B)$ and $S_{vN}(M)$ are constant, with $S_{vN}(B) = S_{vN}(M) \equiv S_0$.

Step 4: Joint entropy increases monotonically. The initial pure state has $S_{vN}(BM) = 0$. Under the Lindblad dynamics, dephasing increases $S_{vN}(BM)$ monotonically from 0 toward the entropy of the fully diagonal state. Since the populations are $(\cos^2 \theta, 0, 0, \sin^2 \theta)$, the late-time diagonal state has $S_{vN}(BM) \rightarrow H_{\text{bin}}(\cos^2 \theta) = S_0$. Therefore $S_{vN}(BM)(t) \in [0, S_0]$ for all t .

Step 5: The ratio r is bounded below by 1. Using $I_{vN} = S_{vN}(B) + S_{vN}(M) - S_{vN}(BM) = 2S_0 - S_{vN}(BM)$:

$$r(t) = \frac{2S_0 - S_{vN}(BM)(t)}{S_0} = 2 - \frac{S_{vN}(BM)(t)}{S_0} \geq 2 - 1 = 1, \quad (17)$$

since $S_{vN}(BM)(t) \leq S_0$. The ratio decreases monotonically from $r(0) = 2$ toward $r(\infty) = 1$. \square

Remark 1. *This result extends beyond the specific initial states considered here. Any Lindblad dynamics that preserves the diagonal populations of ρ_{BM} in the computational basis while increasing $S_{vN}(BM)$ will maintain $r(t) \geq 1$ provided the initial state satisfies $r(0) \geq 1$. The essential mechanism is that the marginals ρ_B and ρ_M are frozen, so I_{vN} can only decrease as the joint state decoheres—but not below $S_{vN}(B)$.*

6 Discussion

6.1 Status of the conjecture

The Born–Fisher–Experiential conjecture (conjecture 1) is cleanly falsified for the qubit toy model class studied here. The falsification is twofold:

1. **Static (Test A):** In the fully decohered diagonal-state regime, the half-saturation tracking accuracy $\alpha_{1/2}(p)$ varies with p but Born-rule body probabilities are indistinguishable from non-Born alternatives.
2. **Dynamic (Test B):** Under Lindblad evolution with exchange coupling and dephasing, the experiential density is non-positive throughout the evolution, making the trajectory functional μ_Q identically zero with no selection mechanism.

6.2 What is not falsified

The experiential density functional ρ_Q itself is well-defined and well-behaved. It correctly identifies the parabolic peak structure ($\rho_Q^{\max} = S_{\text{vN}}(B)/4$ at $r = 1/2$) in the static diagonal-state regime. The issue is not with the functional but with the conjecture that Born-rule probabilities are selected by it. In the dynamical setting, the system never enters the regime where $\rho_Q > 0$, so the functional has no opportunity to act as a selector.

The pure-state negativity $\rho_Q < 0$ for all entangled pure states (eq. (3)) is a structural property of the naive quantum extension, not specific to the Born–Fisher conjecture. Any future quantum application of the experiential density would need to either modify the functional form—for example, replacing I_{vN} with a quantity bounded by $S_{\text{vN}}(B)$ such as the Holevo quantity or accessible information—or restrict the domain to decohered (mixed) states where $r \leq 1$. This structural limitation motivates the search for alternative quantum extensions of the experiential density.

Structurally, the experiential density is parallel to Tononi’s integrated information Φ [14]: both are bipartite information functionals that quantify meaningful internal structure. Quantum extensions of integrated information theory [15] face analogous challenges with entangled states, where quantum correlations inflate the information measures beyond their classically meaningful ranges.

The Lipschitz stability of the classical experiential measure [9] is also unaffected. The classical functional $\rho = I(B; M)(1 - I/H(B))$ retains its properties; the quantum extension simply reveals that the half-saturation condition is not reached during Markovian Lindblad evolution with exchange coupling.

6.3 Future directions

The key question is whether there exist quantum channels for which $r = I_{\text{vN}}/S_{\text{vN}}(B)$ transiently dips below 1, creating a positive ρ_Q window. Several avenues merit investigation:

- **Non-Markovian channels.** Memory effects can produce oscillations in $I_{\text{vN}}(t)$ that might temporarily reduce r below 1. Nakajima–Zwanzig or process-tensor approaches could test this.
- **Amplitude damping.** Unlike dephasing, amplitude damping changes the populations of ρ_B , potentially creating transient under-correlation.
- **Weak coupling.** The exchange Hamiltonian produces perfect tracking ($r \rightarrow 1$) at late times. A weaker or imperfect coupling that allows $r < 1$ could open the experiential-density window.
- **Higher dimensions.** While the $d = 3$ spot check shows the same qualitative behaviour, larger Hilbert spaces may exhibit richer dynamics.

The connection to the Baez–Dolan groupoid cardinality [1], which motivates the $1/|\text{Aut}(x)|$ weighting on the structure space, remains conceptually interesting even though the specific half-saturation conjecture fails. The experiential density framework may find its quantum application in a different variational principle.

7 Conclusion

We have tested the Born–Fisher–Experiential conjecture using a qubit toy model with two complementary approaches: static diagonal-state analysis and full Lindblad master equation dynamics. Both tests yield clear negative results.

The static test shows that the half-saturation tracking accuracy $\alpha_{1/2}(p)$ varies with body probability p (spread 0.069 for $d = 2$), but Born-rule probabilities display no special behaviour compared with non-Born alternatives.

The dynamical test, spanning 1 900+ Lindblad trajectories with symmetric dephasing, asymmetric dephasing, and qutrit extensions, shows that the experiential density $\rho_Q(t) \leq 0$ throughout the entire evolution. The information ratio $I_{\text{vN}}/S_{\text{vN}}(B)$ monotonically decreases from 2 to 1, never entering the under-correlated regime where $\rho_Q > 0$. The trajectory functional $\mu_Q(\theta) = 0$ identically (to 10^{-15} nats·time), with no dependence on initial state, dephasing rate, or coupling strength.

The physical mechanism is the exchange Hamiltonian’s perfect-tracking behaviour: the system transitions directly from over-correlated (entangled) to perfectly correlated (classical tracking) without passing through an under-correlated phase. The conjecture is cleanly falsified for this model class. Non-Markovian channels and amplitude damping remain open avenues for modified conjectures.

A Numerical Validation

All computations were implemented in Python (NumPy) without external quantum libraries. We report two suites of validation checks: 10 quantum-information sanity checks for the core utilities and 6 Lindblad integrator validation checks.

A.1 Quantum information sanity checks

#	Check	Expected	Result
(a)	$\text{Tr}(\rho) = 1$, PSD for $\rho_{BM}(0.5, 0.75)$	pass	pass
(b)	Pure Bell state $I_{\text{vN}} = 2 \ln 2$	1.38629 nats	1.38629 nats ($\delta < 10^{-12}$)
(c)	Product state $I_{\text{vN}} = 0$	0	$< 10^{-12}$
(d)	$S_{\text{vN}}(I/2) = \ln 2$	0.69315 nats	0.69315 nats ($\delta < 10^{-12}$)
(e)	$S_{\text{vN}}(I/3) = \ln 3$	1.09861 nats	1.09861 nats ($\delta < 10^{-12}$)
(f)	Araki–Lieb bound, 10 random states	$0 \leq I_{\text{vN}} \leq 2 \min(S_{\text{vN}}(B), S_{\text{vN}}(M))$	10/10 pass
(g)	Symmetry $I(p, \alpha) = I(1 - p, \alpha)$	$\delta < 10^{-12}$	max err $< 2 \times 10^{-12}$
(h)	Eigenvalue cutoff stability ($\varepsilon = 10^{-15, -14, -13}$)	agreement	$\delta < 10^{-13}$
(i)	Perfect tracking $\alpha = 1$: $r = 1$	1.0	1.0 ($\delta < 10^{-12}$)
(j)	No tracking $\alpha = 0.5$: $I_{\text{vN}} = 0$	0	$< 10^{-12}$

Table 1: Quantum information utility validation. All 10 checks pass to machine precision or better.

A.2 Lindblad integrator validation

#	Check	Tolerance	Result
(a)	Trace preservation: $\max \text{Tr}(\rho) - 1 $	10^{-10}	$< 10^{-16}$
(b)	Positivity: $\min(\lambda_i)$	$\geq -10^{-10}$	$\geq -2 \times 10^{-31}$
(c)	Pure dephasing rate: $ \rho_{BM}[0, 3](t) $	5% relative	5×10^{-11} relative
(d)	Initial condition: $I_{vN}(0) = 2 \ln 2$	10^{-12}	$\delta < 10^{-12}$
(e)	Late-time diagonal: off-diagonal sum	$< 10^{-4}$	$< 2 \times 10^{-9}$
(f)	RK4 convergence: $ \mu_Q(n=2000) - \mu_Q(n=4000) $	10^{-4}	$< 10^{-15}$

Table 2: Lindblad integrator validation. All 6 checks pass with margins of several orders of magnitude. The trace is preserved to 10^{-16} , positivity to 10^{-31} , and RK4 convergence is verified to $O(h^4)$.

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